The Self-Gravitational Corrections as the Source for Stiff Matter on the Brane in $SAdS_5$ Bulk

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Abstract

A $D3-$brane with non-zero energy density is considered as the boundary of a five dimensional Schwarzschild anti de Sitter bulk background. Taking into account the semi-classical corrections to the black hole entropy that arise as a result of the self-gravitational effect, and employing the AdS/CFT correspondence, we obtain the self-gravitational correction to the first Friedmann-like equation. The additional term in the Hubble equation due to the self-gravitational effect goes as a^{-6}. Thus, the self-gravitational corrections act as a source of stiff matter contrary to standard FRW cosmology where the charge of the black hole plays this role.

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1 Introduction

In recent years, a lot of interest has been raised in studying the cosmology of universes with extra dimensions [1, 2, 3]. The common feature of all these models is the distinction of observable universe (the three-brane)from the rest of the universe (the bulk). Ordinary matter field are assumed to live on the brane while gravity propagates in the whole space-time. The braneworld scenario gained momentum as a solution to the hierarchy problem [1, 2, 3]. The holographic principle, meanwhile, was first realized in string theory via the AdS/CFT correspondence[4, 5, 6]. The essence of braneworld holography [7]-[10] can be captured in the following claim : Randall-Sundrum branworld gravity is dual to a CFT with a UV cutoff, coupled to gravity on the brane. Formal evidence for this claim was provided by studying a brane universe in the background of the Schwarzschild-AdS Bblack hole. The introduction of the black hole on the gravity side of the AdS/CFT correspondence corresponds to considering finite temperature states in the dual CFT [11]. In the context of braneworld holography, Savonije and Verlinde demonstrated that their induced braneworld cosmology could alternatively be described as the standard FRW cosmology driven by the energy density of this dual CFT [12, 8].

Black hole thermodynamic quantities depend on the Hawking temperature via the usual thermodynamic relations. The Hawking temperature undergoes corrections from many sources: the quantum corrections [13]- [19], the self-gravitational corrections [20, 21], and the corrections due to the generalized uncertainty principle [22, 23]. Concerning the quantum process called Hawking effect [24] much work has been done using a fixed background during the emission process. The idea of Keski-Vakkuri, Kraus and Wilczek (KKW) [20] is to view the black hole background as dynamical by treating the Hawking radiation as a tunnelling process. The energy conservation is the key to this description. The total (ADM) mass is kept fixed while the mass of the black hole under consideration decreases due to the emitted radiation. The effect of this modification gives rise to additional terms in the formulae concerning the known results for black holes [21]; a nonthermal partner to the thermal spectrum of the Hawking radiation shows up.

In the present paper, we take into account corrections to the entropy of the five-dimensional Schwarzschild- anti de Sitter black hole (abbreviated to $SAdS_5$ in the sequel) that arise due to the self-gravitational effect. Previous studies of the Cardy-Verlinde formula (or the corresponding Friedmann equation) in an AdS/CFT context have attracted a lot of attention [25, 26]. In the previous paper [27], by studying the case of an empty brane with zero cosmological constant, embedded in an $SAdS_5$ black hole bulk spacetime, and introducing the self-gravitational correction to the Cardy-Verlinde formula, we were able to find a host of interesting cosmological solutions for the brane universe. The self-gravitational correction, acts as a source for stiff matter on the brane, whose equation of state is simply given by the pressure being equal to the energy density. Due to the self-gravitational corrections, a bouncing universe could, arise, i.e. a universe that bounce from a contracting phase to an expanding one without encountering a singularity.

Previously have been shown that the charged Ads/CFT black hole background provides support for a singularity-free cosmology [28]-[33] in which the big Bang singularity is not present. In the present paper we consider the more physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane.

2 Self-Gravitational Corrections to FRW Brane Cosmology

In the asymptotic coordinates, the $SAdS_5$ black hole metric is

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega_{(3)}^2, \tag{1}$$

where

$$F(r) = 1 - \frac{\mu}{r^2} + r^2, \tag{2}$$

and $l = 1$ is the AdS radius . The parameter μ is proportional to the ADM mass M of the black hole.

Due to the self-gravitational corrections, the modified Cardy-Verlinde formula for the entropy of the $SAdS_5$ black hole is given as [27]

$$S_{CFT} = \frac{2\pi r}{3} \sqrt{\left| \left[E_C - \frac{1}{r}\omega \right] \left[(2E_4 - E_C) - \frac{1}{r}\omega \right] \right|} \tag{3}$$

and keeping terms up to first order in the emitted energy ω, it takes the form

$$S_{CFT} = s_{CFT} (1 - \varepsilon\omega) \tag{4}$$

where the small parameter ε is given by

$$\varepsilon = \frac{1}{r} \frac{E_4}{E_C (2E_4 - E_C)} . \tag{5}$$

where E_C is the Casimir energy, the four-dimensional energy E_4, is given by

$$E_4 = \frac{l}{r} E \tag{6}$$

where E is the thermodynamical energy of the black hole.

We now consider a 4-dimensional brane in the $SAdS_5$ black hole background. This 4-dimensional brane can be regarded as the boundary of the 5-dimensional $SAdS_5$ bulk background. Let us first replace the radial coordinate r with a and so the line element (1)

$$ds^2 = -F(a)dt^2 + \frac{1}{F(a)}da^2 + a^2 d\Omega_{(3)}^2, \tag{7}$$

Within the context of the AdS/CFT correspondence, Savonije and Verlinde studied the CFT/FRW-cosmology relation from the Randall-Sundrum type braneworld perspective [8]. They showed that the entropy formulas of the CFT coincides with the Friedmann equations when the brane crosses the black hole horizon.

In the case of a 4-dimensional timelike

$$ds_{(4)}^2 = -d\tau^2 + a^2(\tau)d\Omega_{(3)}^2 , \tag{8}$$

One of the identifications that supports the CFT/FRW-cosmology relation is

$$H^2 = \left(\frac{2G_4}{V}\right)^2 S^2 \tag{9}$$

where H is the Hubble parameter defined by $H = \frac{1}{a}\frac{da}{d\tau}$ and V is the volume of the 3-sphere ($V = a^3 V_3$). The 4-dimensional Newton constant G_4 is related to the 5−dimensional one G_5 by

$$G_4 = \frac{2}{l}G_5 \ . \tag{10}$$

It was shown that at the moment that the 4-dimensional timelike brane crosses the cosmological horizon, i.e. when $a = a_b$, the CFT entropy and the entropy of the $SAdS_5$ black hole are identical. By substituting (4) into (9), we obtain the self-gravitational corrections to the motion of the CFT-dominated brane

$$H^2 = \left(\frac{2G_4}{V}\right)^2 S^2_{CFT} \left(1 - \varepsilon\omega\right)^2 \ . \tag{11}$$

It is obvious that from the first term on the right-hand side of (11) we get the standard Friedmann equation with the appropriate normalization

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho \tag{12}$$

where ρ is the energy density defined by $\rho = E_4/V$, and k taking values $+1, 0, -1$ in order to describe, respectively, the elliptic, flat, and hyperbolic horizon geometry of the $SAdS_5$ bulk black hole. If we consider the more physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane, then the first Friedmann equation takes the following form

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho - \frac{1}{l^2} + \frac{4\pi}{3M_p^2 \rho_0}(\rho_0 + \rho_{br})^2 \tag{13}$$

where ρ_0 is the tension of the brane, while ρ_{br} is the energy density of radiation. The Hubble equation can be rewritten as

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho + \frac{\Lambda_4}{3} + \frac{8\pi}{3M_p^2}(\frac{\rho_{br}^2}{2\rho_0} + \rho_{br}) \tag{14}$$

where

$$\Lambda_4 = \frac{4\pi\rho_0}{M_p^2} - \frac{3}{l^2} = \Lambda_{br} - \frac{3}{l^2} \tag{15}$$

is the effective cosmological constant of the brane. The correction to the FRW equation due to the self-gravitation effect is expressed by the second term in the right-hand side of Eq.(11). Keeping terms up to first order in the emitted energy ω, the modified Hubble equation due to the self-gravitation correction is

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho + \frac{\Lambda_4}{3} + \frac{8\pi}{3M_p^2}(\frac{\rho_{br}^2}{2\rho_0} + \rho_{br}) - \frac{8\pi G_4}{3}\left[\frac{4\pi G_4}{3}\frac{1}{a^2 V_3}\rho\right]\omega \tag{16}$$

By tuning the bulk cosmological constant and the brane tension Λ_{br}, the effective four dimensional cosmological constant Λ_4 can be set to zero, here we would like consider this

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critical brane. Also for the very small a the curvature term $\frac{-k}{a^2}$ can be neglected in the above equation relative to the other contributors. The radiation energy density ρ_{br} on the brane is as

$$\rho_{br} = \frac{\rho_r}{a^4} \tag{17}$$

where ρ_r is a constant, then we can rewrite the cosmological equation (16) as

$$H^2 = (\epsilon_3 M + \frac{8\pi\rho_r}{3M_p^2})\frac{1}{a^4} - \frac{\epsilon_3^2 M\omega}{2a^6} + \frac{4\pi\rho_r^2}{3M_p^2\rho_0 a^8} \tag{18}$$

where

$$\epsilon_3 = \frac{16\pi G_5}{3V_3} \tag{19}$$

The evolution of the system can be solved exactly, as one can most simply realize by using conformal time η, defined as $dt = a(\eta)^2 d\eta$ and new variable $x = a^2$:

$$\frac{1}{4}x'^2 = c_1 x^2 + c_2 x + c_3 \tag{20}$$

where

$$c_1 = \epsilon_3 M + \frac{8\pi\rho_r}{3M_p^2} \tag{21}$$

$$c_2 = \frac{-\epsilon_3^2 M\omega}{2} \tag{22}$$

$$c_3 = \frac{4\pi\rho_r^2}{3M_p^2\rho_0} \tag{23}$$

With the assumption that a bounce does indeed take place, and setting $x' = 0$, the following condition emerges

$$\frac{\epsilon_3 M^2\omega^2}{4} \geq \frac{16\pi\rho_r^2}{3M_p^2\rho_0}(\epsilon_3 M + \frac{8\pi\rho_r}{3M_p^2}) \tag{24}$$

The solution of Eq.(20) is as

$$a^2 = \frac{-c_2}{2c_1} + \sqrt{\Delta}\cosh(2\sqrt{c_1}\eta) \tag{25}$$

where

$$\Delta = \frac{-c_3}{c_1} + (\frac{c_2}{c_1})^2 > 0. \tag{26}$$

The time variables are related as

$$t = \frac{-c_2}{2c_1}\eta + \sqrt{\frac{\Delta}{4c_1}}\sinh(2\sqrt{c_1}\eta) \tag{27}$$

We set $\eta = 0$ at the bounce, then t is also vanish at the bounce. Therefore the minimal value of the scale factor is given by

$$a_{min}^2 = \frac{-c_2}{2c_1} + \sqrt{\Delta}. \tag{28}$$

An immediate consequence of last term in Eq.(18) is that the bounce can only occur if self-gravitational corrections satisfy condition (24). In the other hand, for an empty brane, a bounce obtain for any value of the self-gravitational corrections.

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3 Conclusions

In this paper we have considered a four-dimensional timelike brane with non-zero energy density as the boundary of the $SAdS_5$ bulk background. Exploiting the CFT/FRW-cosmology relation, we have derived the self-gravitational corrections to the first Friedmann-like equation which is the equation of the brane motion. The additional term that arises due to the semiclassical analysis, can be viewed as stiff matter where the self-gravitational corrections act as the source for it. This result is contrary to standard analysis that regards the charge of $SAdS_5$ bulk black hole as the source for stiff matter. In previous paper [27] we have considered an empty critical brane. The effective cosmological constant Λ_4 on the brane gets a contribution from the bulk cosmological constant and the brane tension. For the critical brane we finely tune Λ_4 to zero. In the non-critical empty brane case, the solution of FRW equation at very small value of the scale factor, are close to the behaviour for critical brane case. The cosmological constant term dominant at large value of scale factor, where the stiff matter term becomes irrelevant. In this paper we have focused on the critical non-empty brane case. This is more physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane. For small scale factor a we have neglected the curvature term $\frac{-k}{a^2}$. Similar to the empty brane case even if we consider the non-zero cosmological constant term (non-critical brane case) for the small scale factor, this is irrelevant. The first Friedmann equation gets a contribution proportional to a^{-6} due to the self-gravitational corrections with an unconventional negative sign. This term behave like the stiff matter on the brane. It is obvious that it is dominant at early time of the brane evolution, also have interesting cosmological consequences. The bounce can be attributed to the negative-energy matter, which dominates at small values of a and create a significant enough repulsive force so that a big crunch is avoided.

In the empty brane case, a bounce will be obtained due to the self-gravitational corrections, regardless of how small, but in the non-empty brane case the bounce can only occur if the self-gravitational correction satisfies the condition (24).

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